

Los Alamos National Laboratory,  
February 3-5, 2003

# Dissipative solitons

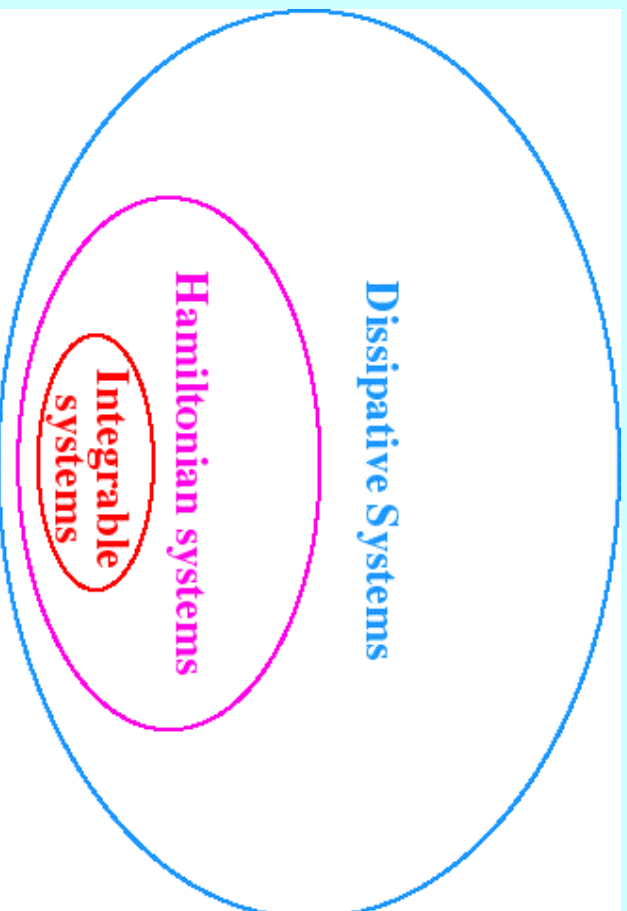
**Nail Akhmediev**

Optical Sciences Center, Research School of Physical Sciences and Eng., Institute of Advanced Studies, the Australian National University, Canberra, ACT 0200, Australia

## **Abstract**

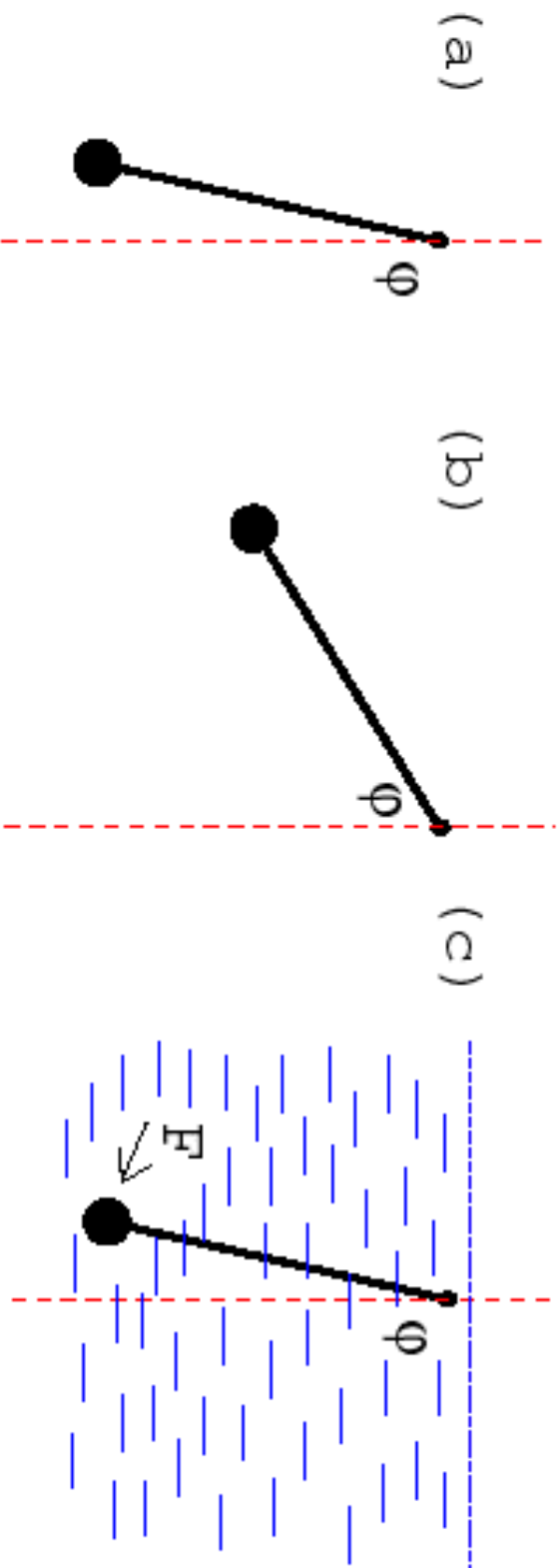
The concept of the soliton has been changing over the last decade or two. It changed from being a nonlinear mode of an integrable system to being described as a Hamiltonian soliton, and moreover, a dissipative soliton. The latter notion includes ultra-short pulses generated by laser systems, spatial solitons in wide aperture lasers, dispersion managed solitons in all-optical systems with gain and loss and many other similar applications. This talk is aimed to reflect this new items in the general list of solitons and to discuss their common features and differences from the traditional concept of solitons.

# **A Rough Classification of Nonlinear Systems with an Infinite Number of Degrees of freedom admitting soliton solutions**



**Hamiltonian Systems  
can be considered as  
a subclass of  
dissipative ones,  
while integrable  
systems can be  
viewed as a subclass  
of Hamiltonian ones.**

# A classification of a dynamical system with one degree of freedom

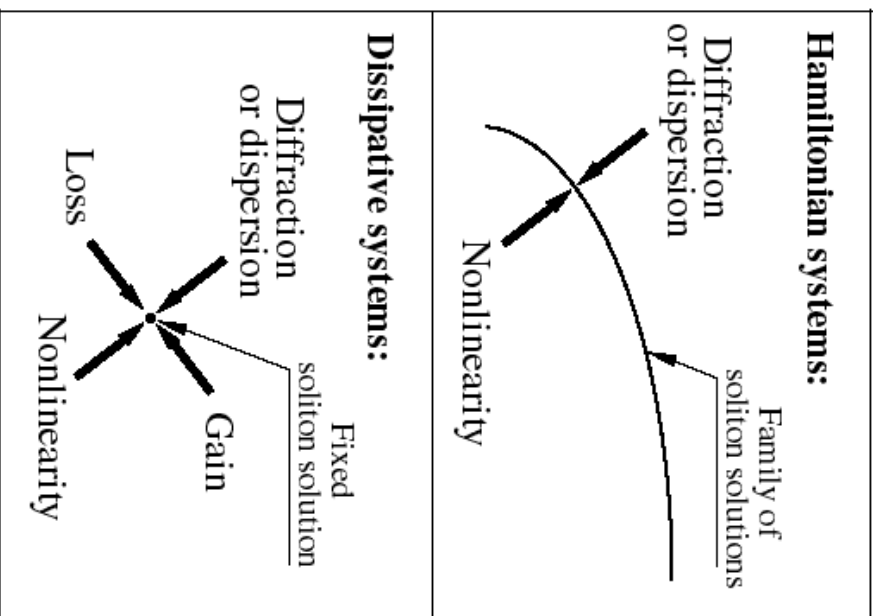


(a) The amplitude is small (linear oscillations)

(b) The amplitude is not small (nonlinear oscillations).

(c) Dissipative system.

# Qualitative difference between the soliton solutions in Hamiltonian and dissipative systems.

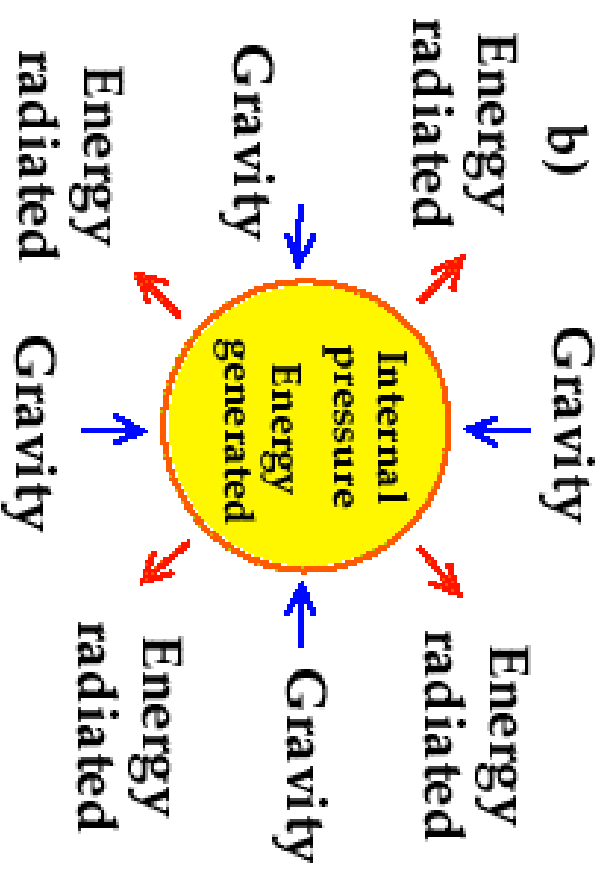
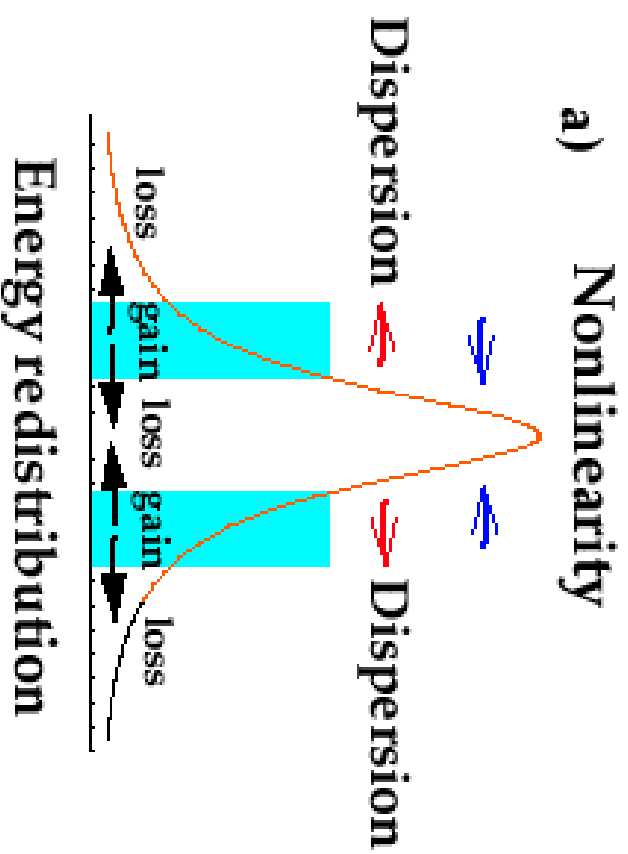


**In Hamiltonian systems, soliton solutions are the result of a single balance, and comprise one- or few-parameter families.**

**In dissipative systems, the soliton solutions are the result of a double balance and, in general, are isolated.**



# Qualitative description of solitons in dissipative systems.



The soliton is a result of a dynamical process of energy exchange with the environment and between its own parts. The areas of consumption and dissipation of energy are both frequency and intensity dependent.

The sun as an example of an object in dynamic equilibrium. Gravitational forces balance the internal pressure and the generated energy is balanced by the emitted radiation.

# Composite solitons in passively mode-locked lasers and the complex quintic Swift-Hohenberg equation

- The complex quintic Swift-Hohenberg equation (CSHE) is a model for describing pulse generation in mode-locked lasers with fast saturable absorbers and a complicated spectral response.
- Using numerical simulations, we study single and two-soliton solutions of the (1+1)-dimensional complex quintic Swift-Hohenberg equations. We have found that several types of stationary and moving composite solitons of this equation are generally stable.

# The complex quintic Swift-Hohenberg equation (CSHE)

Dispersion

Nonlinearity

Higher order

Nonlinearity

$$i\psi_t + \frac{1}{2}\psi_{xx} + |\psi|^2\psi + \nu|\psi|^4\psi =$$

$$i\delta\psi + i\varepsilon|\psi|^2\psi + i\mu|\psi|^4\psi + i\beta\psi_{xx} + i\gamma_2\psi_{xxxx}$$

Linear Gain

Nonlinear Gain-Loss

Spectral Filtering

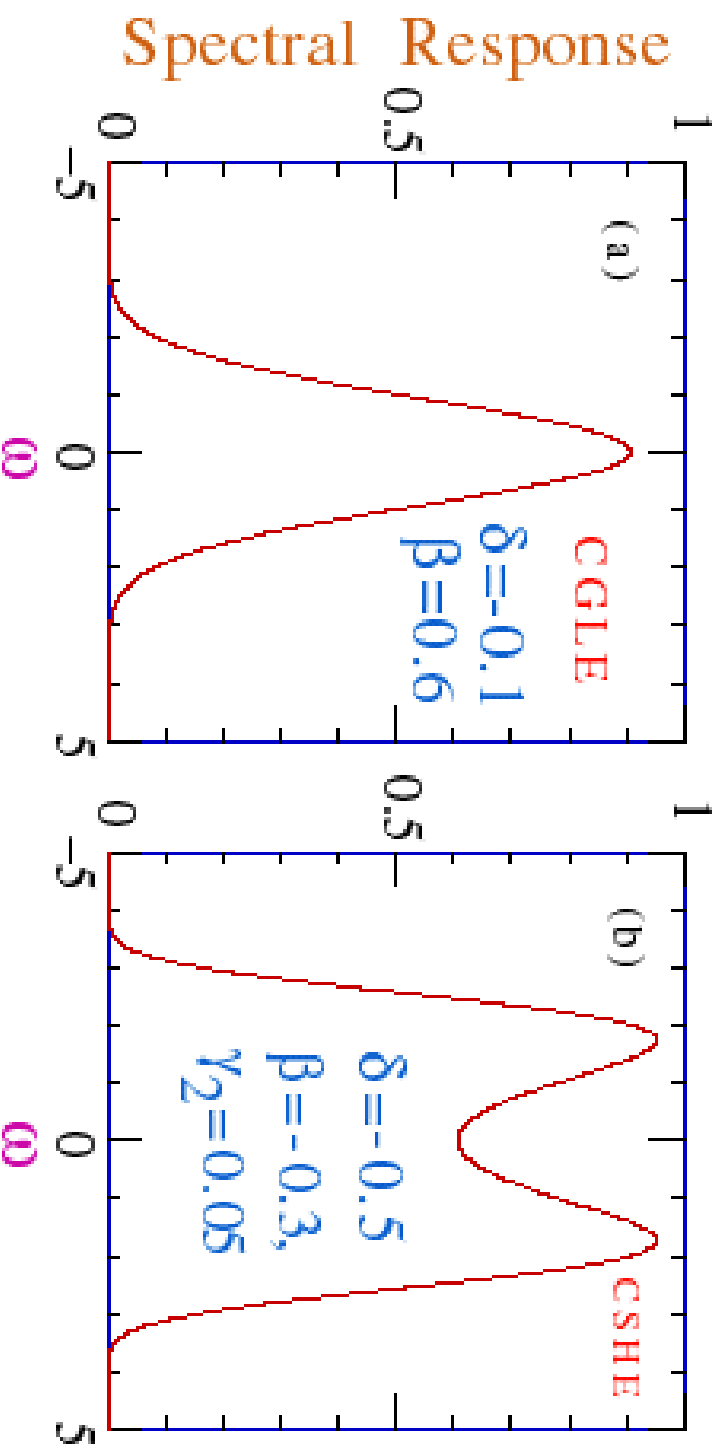
# Energy balance equation

The energy  $Q = \int_{-\infty}^{\infty} |\psi|^2 dx$  is not conserved

We have the energy balance equation instead

$$\frac{dQ}{dt} = 2 \int_{-\infty}^{\infty} \left[ \delta |\psi|^2 - \beta \left| \frac{\partial \psi}{\partial t} \right|^2 - \gamma_2 \left| \frac{\partial^2 \psi}{\partial t^2} \right|^2 + \varepsilon |\psi|^4 + \mu |\psi|^6 \right] dx$$

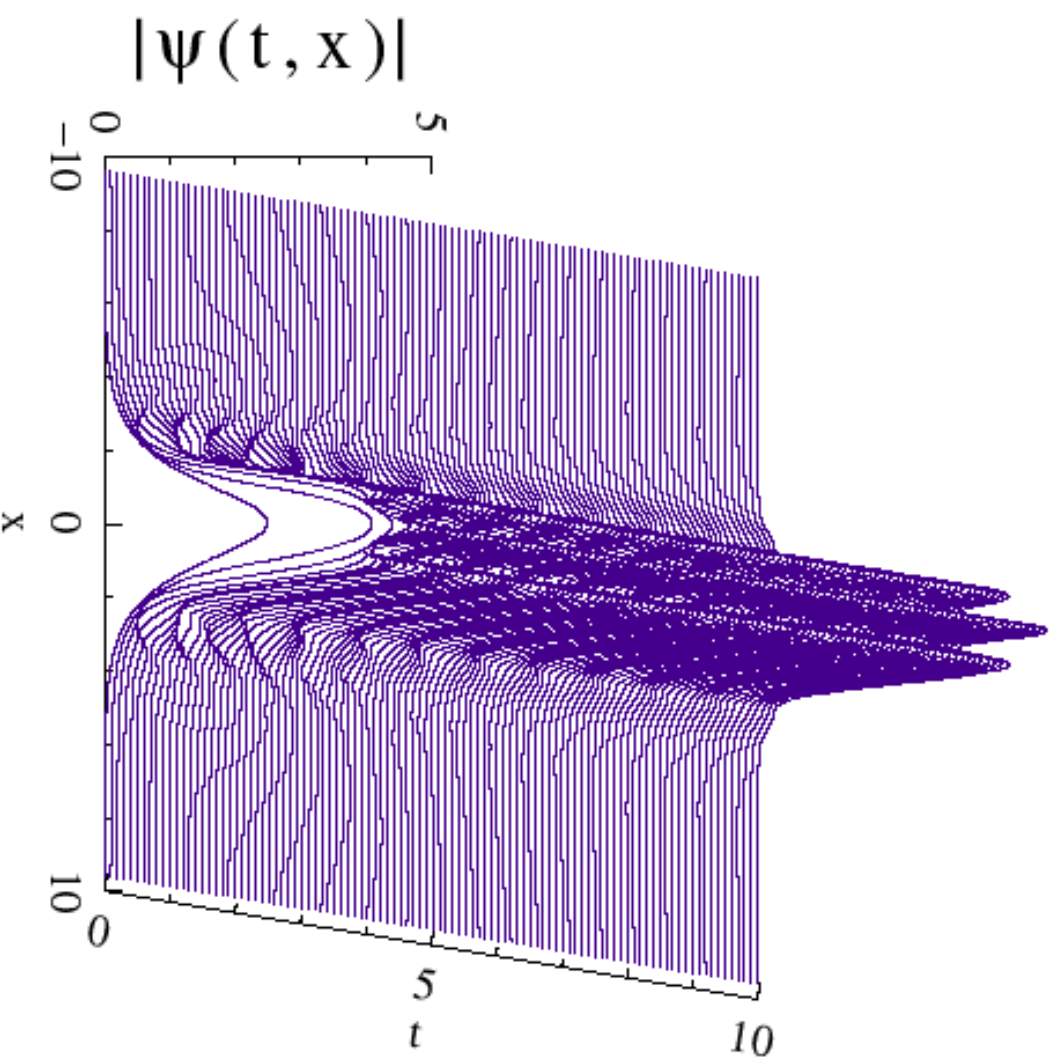
# Spectral filtering in the two models of a laser



**Spectral filtering  $T(\omega) = \exp(\delta - \beta \omega^2 - \gamma_2 \omega^4)$  in the two models:**

**(a) CGL and (b) CSHE. Parameters are: (a)  $\beta = 0.6$ ,  $\gamma_2 = 0$ , and  $\delta = -0.1$ , and (b)  $\beta = -0.3$ ,  $\gamma_2 = 0.05$  and  $\delta = -0.5$ .**

Convergence of a sech-type initial condition to a CP soliton.



Parameters of  
the simulation  
are:

$$\beta = -0.3,$$

$$\beta\gamma_2 = 0.05,$$

$$\varepsilon = 1.6,$$

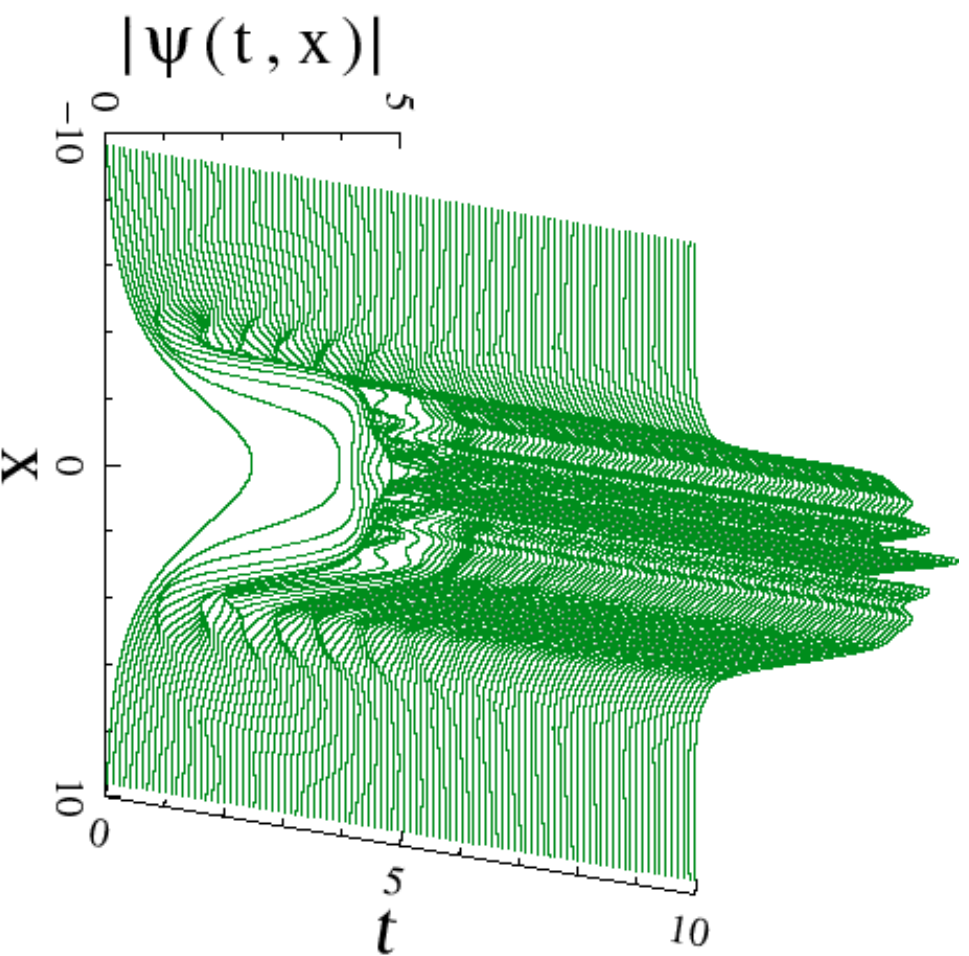
$$\nu = 0,$$

$$\mu = -0.1$$

$$\text{and } \delta = -0.5.$$

$$\text{Here } Q = 38.4.$$

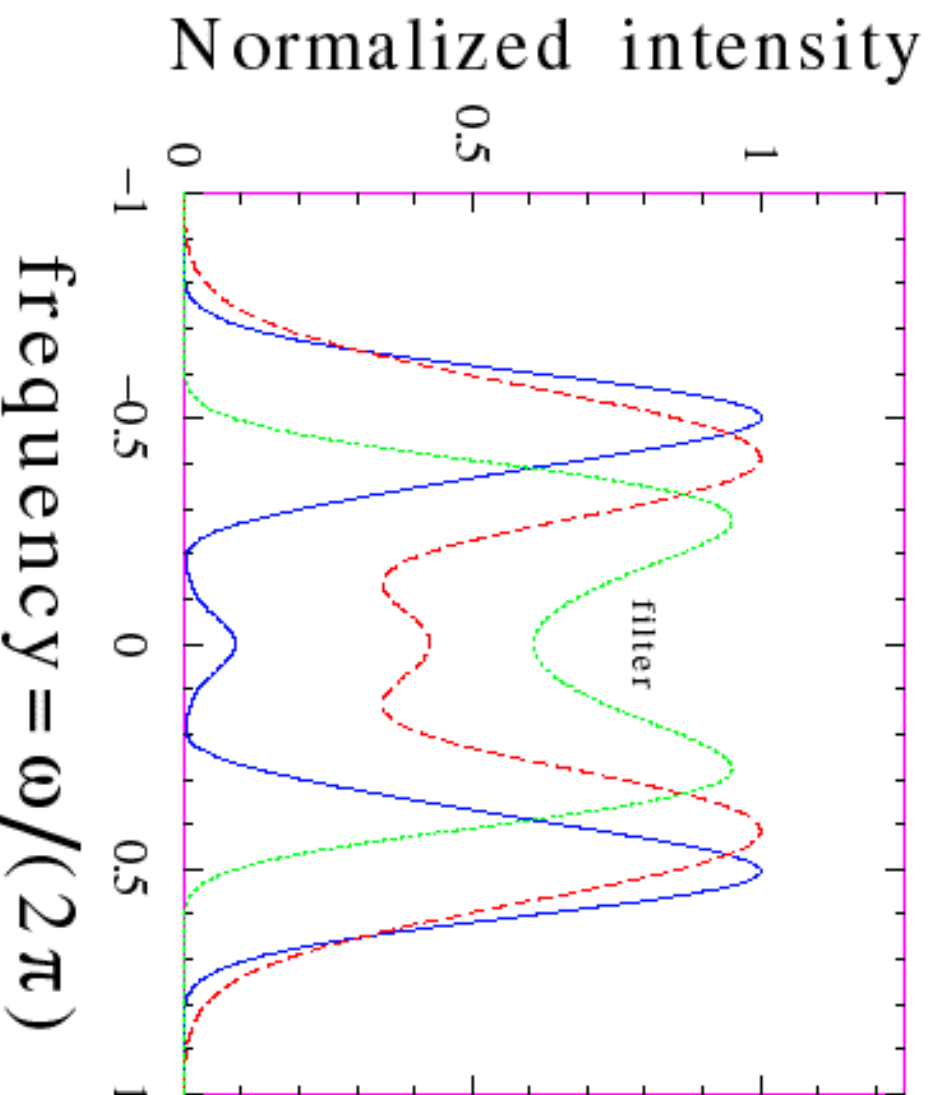
Convergence of a sech-type initial condition to an NCP soliton.



The equation parameters of the simulation are the same as in the previous slide.

Here  $Q=74.0$ .

# Spectra of the two pulses (CP and NCP) and spectral filtering of the CSHE.

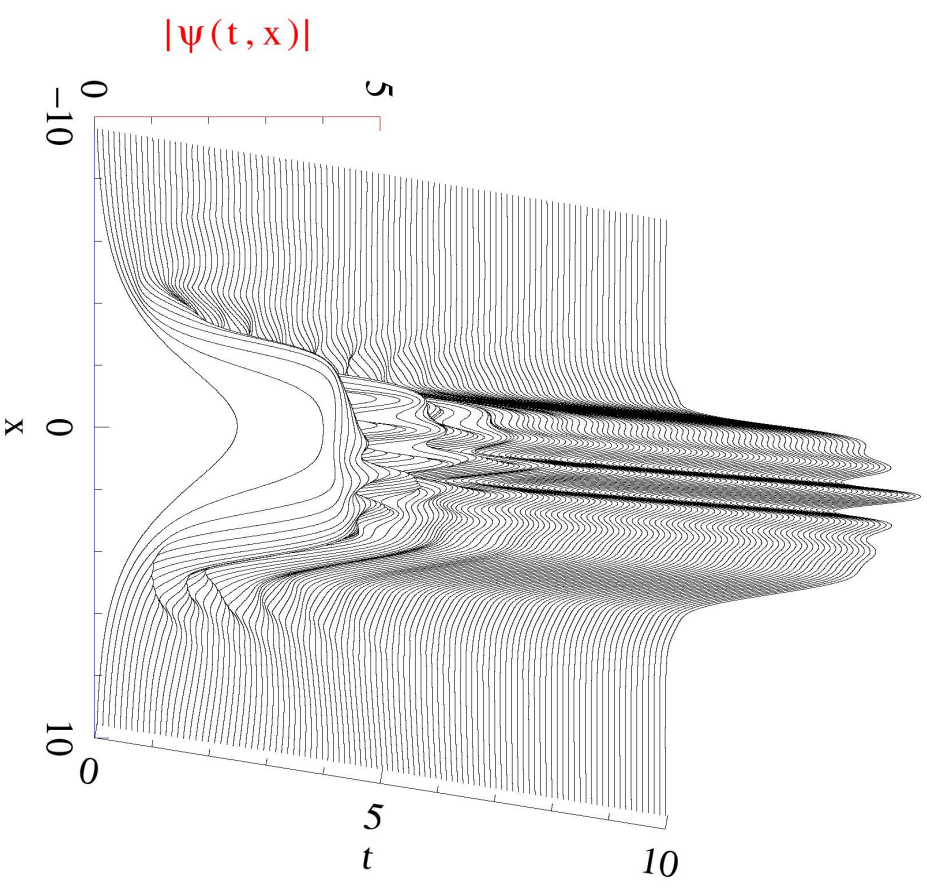
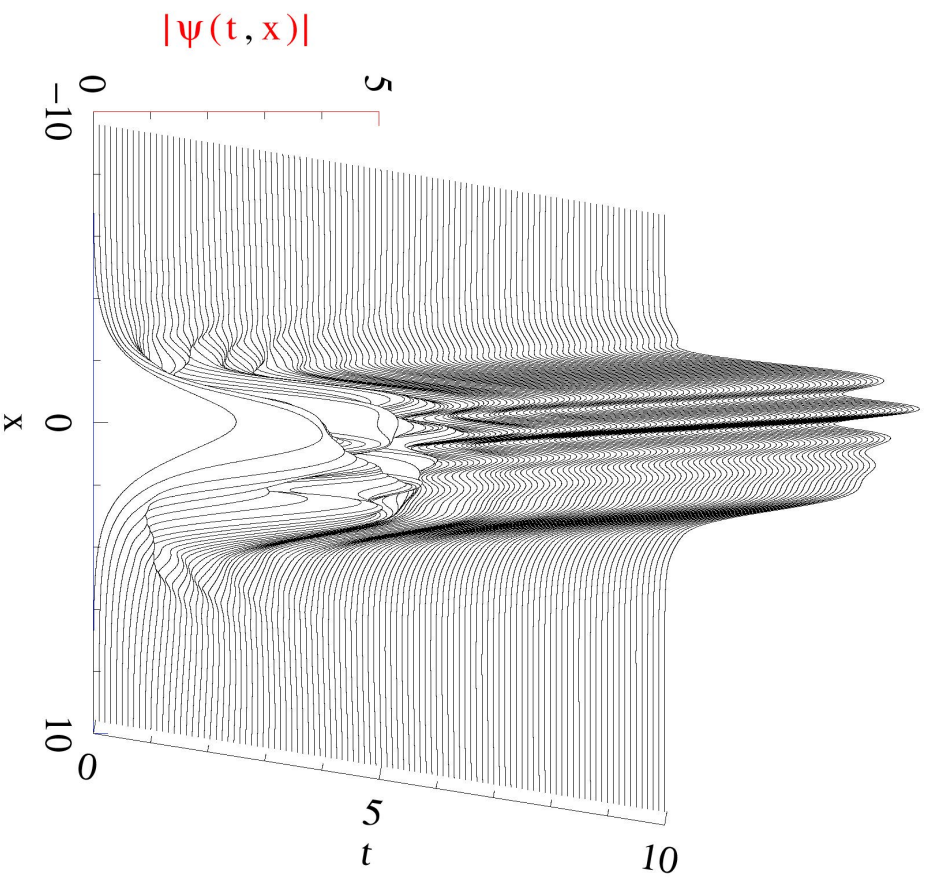


Spectra of the **CP** (red line) and **NCP** (blue line).

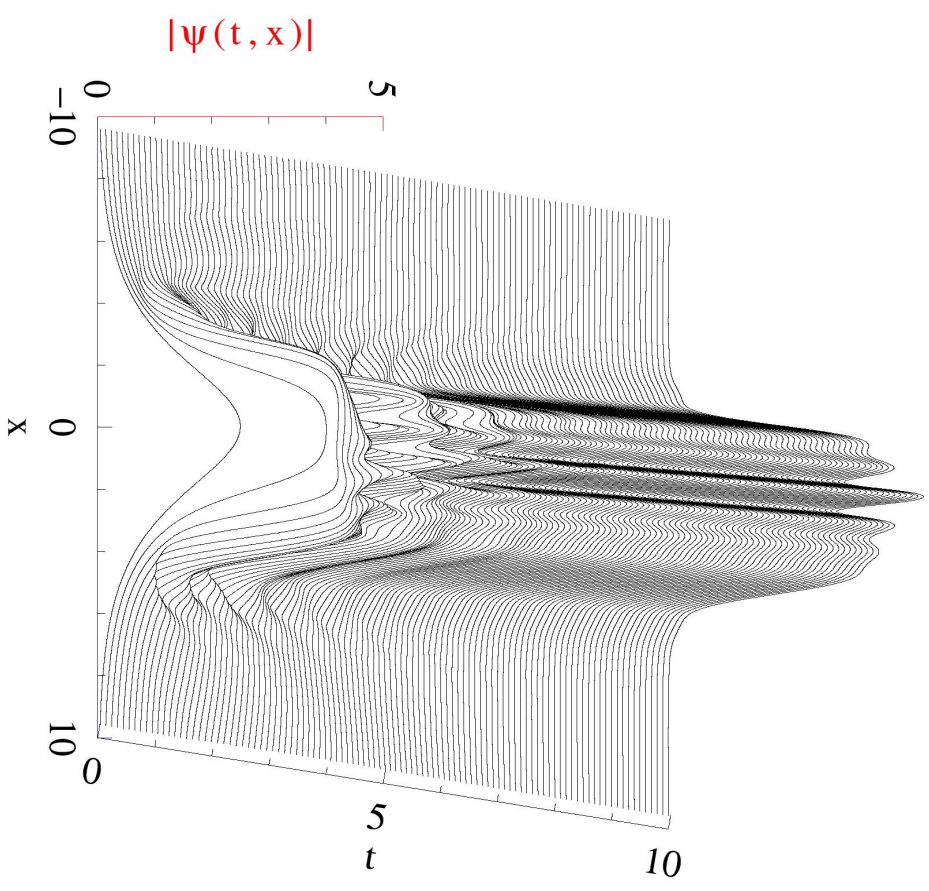
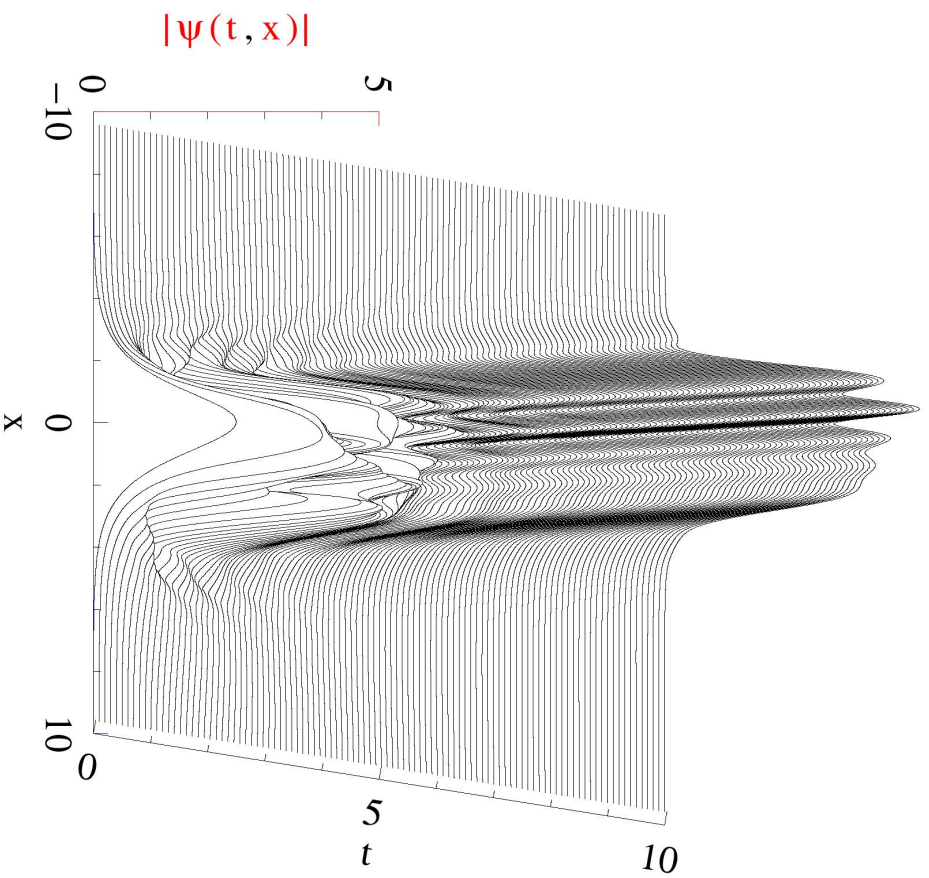
Spectral filtering of the **CSHE** is shown by the green line.



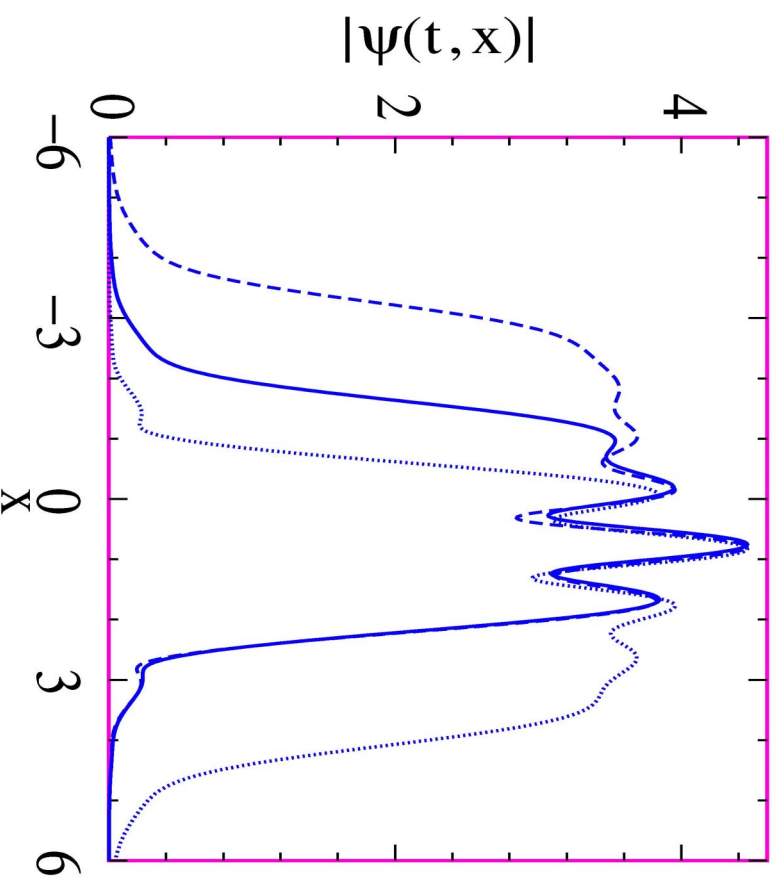
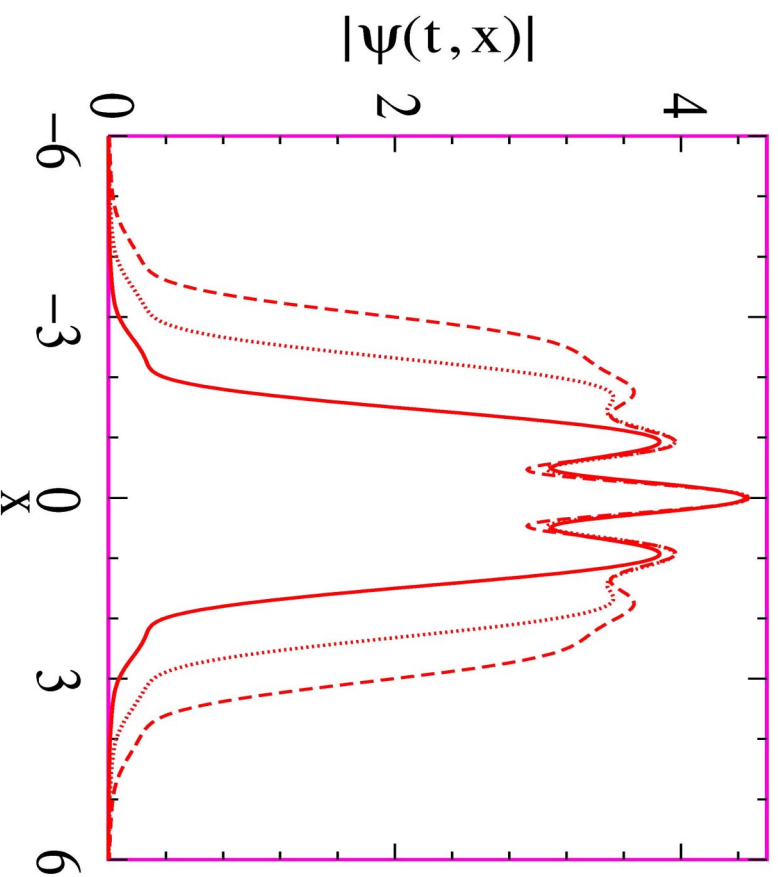
# Two examples of an asymmetric moving pulse



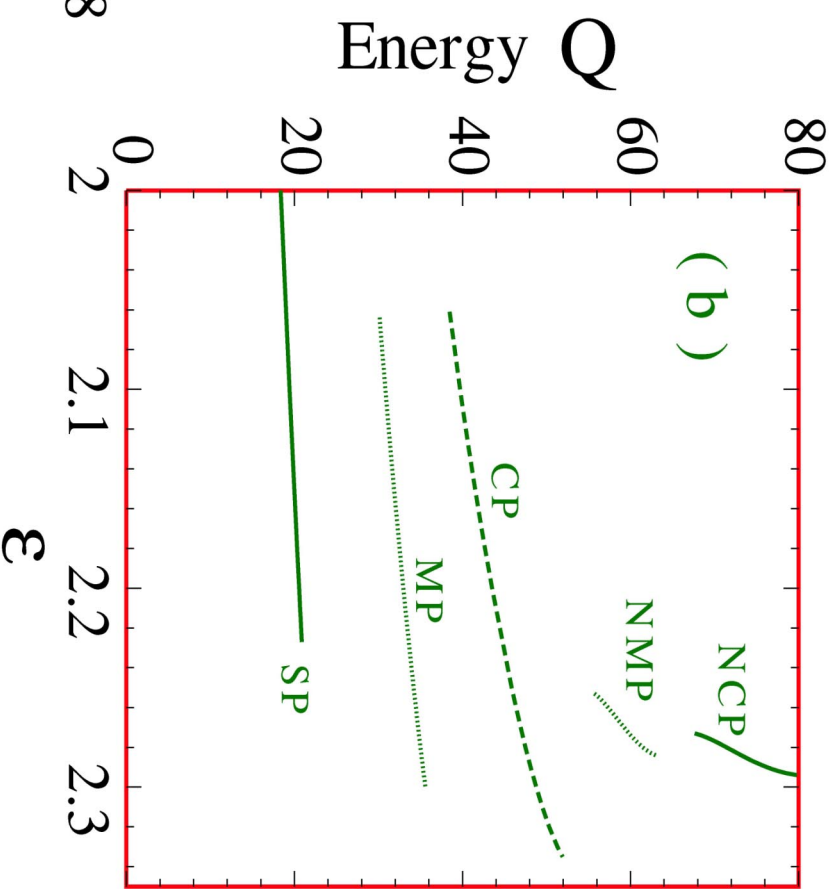
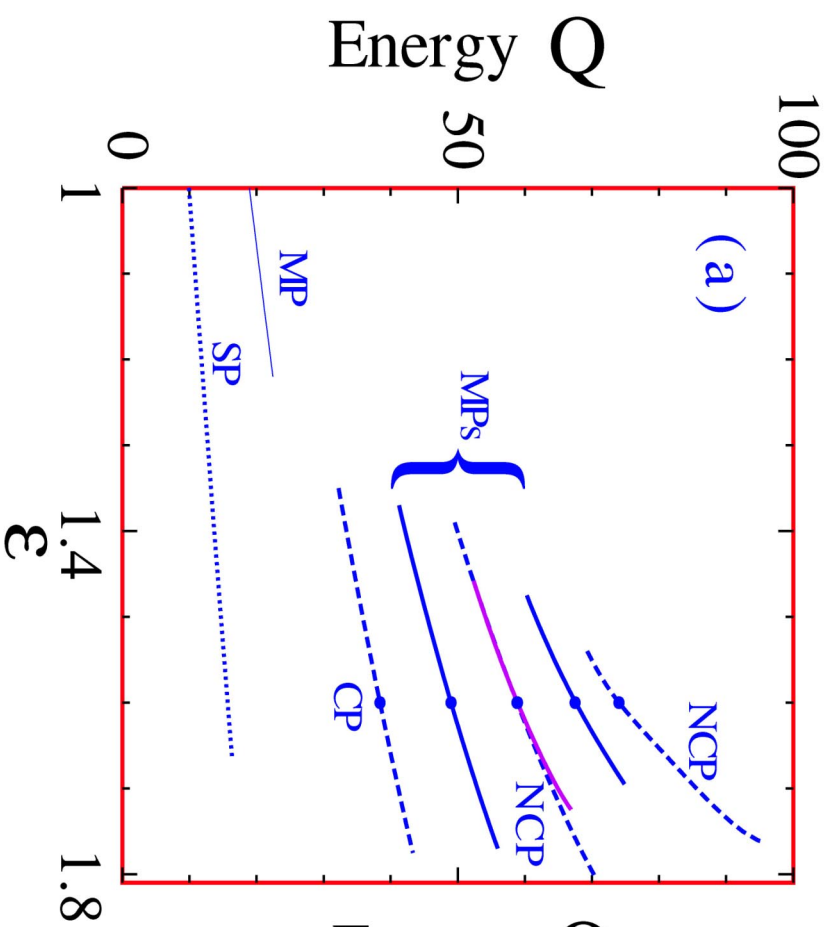
# Two examples of an asymmetric moving pulse



Various soliton profiles,  
symmetric and asymmetric  
(moving solitons).

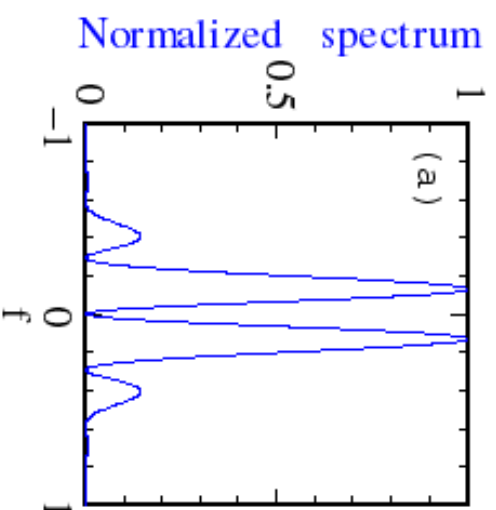


Various soliton branches in the  $Q - \epsilon$  plane in the case of (a) CSH equation and (b) CGL equation

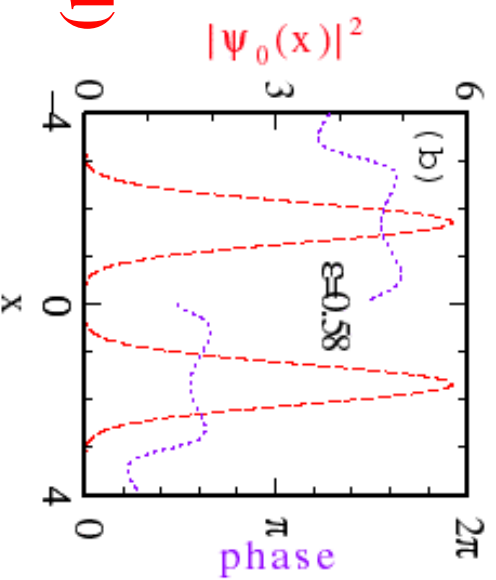


# Type I and type II double pulse solutions.

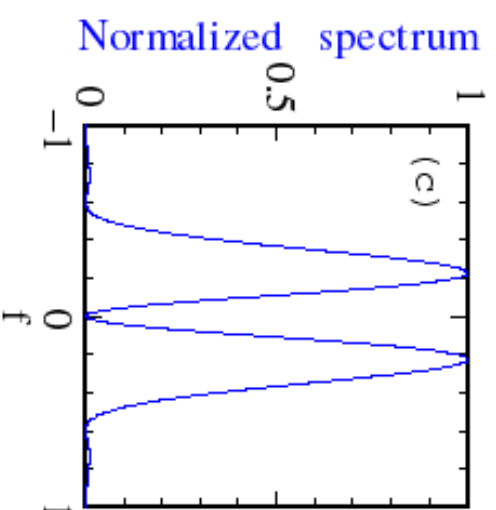
**Spectrum  
(blue)**



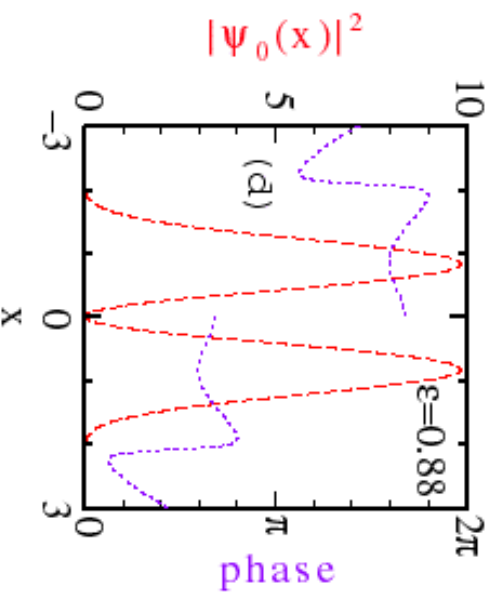
**Type I  
(left)**



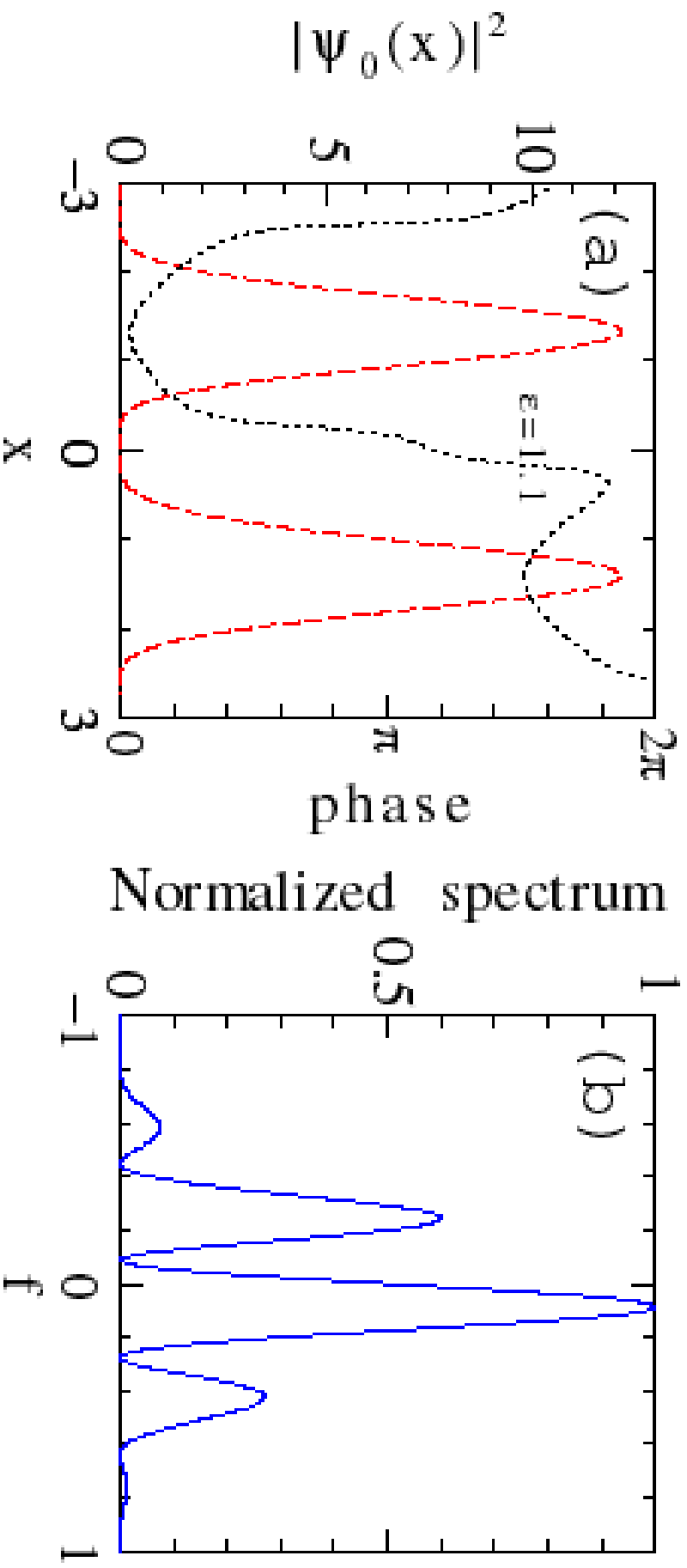
**Spectrum  
(blue)**



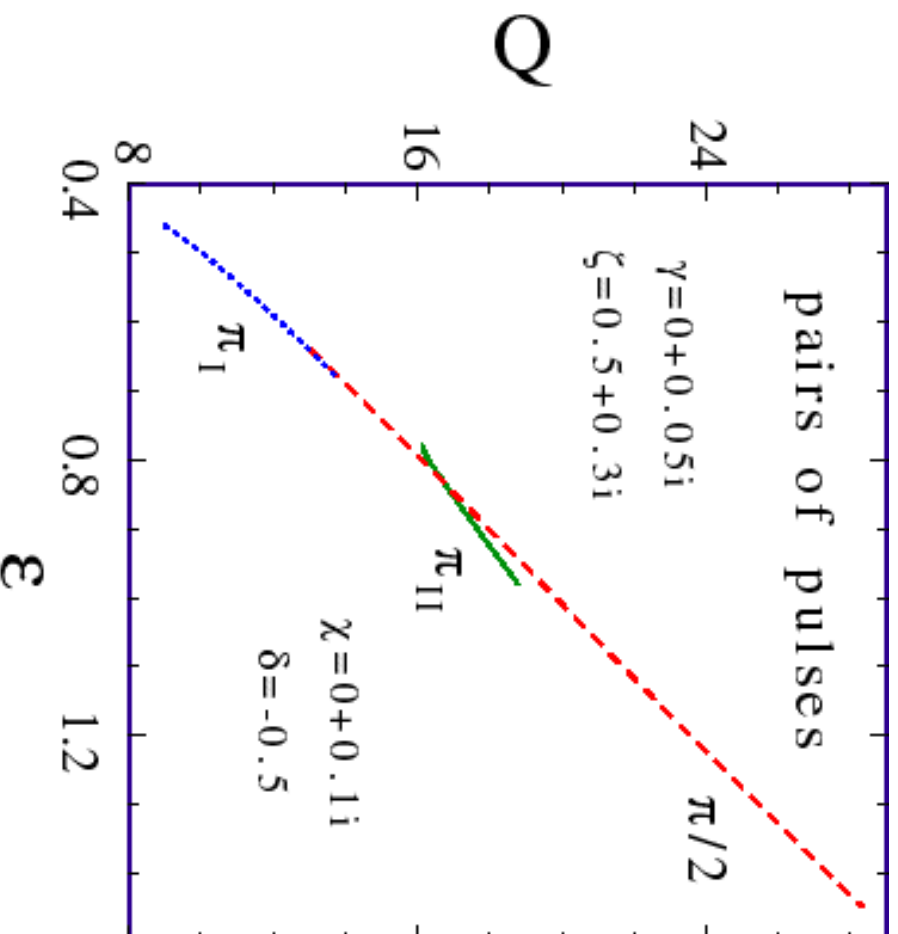
**Type II  
(right)**



**The intensity and phase profiles.**  
 The spectrum of the double pulse  
 solutions with  $\pi/2$  phase difference.



# Energy versus $\varepsilon$ for three types of pulse pairs.



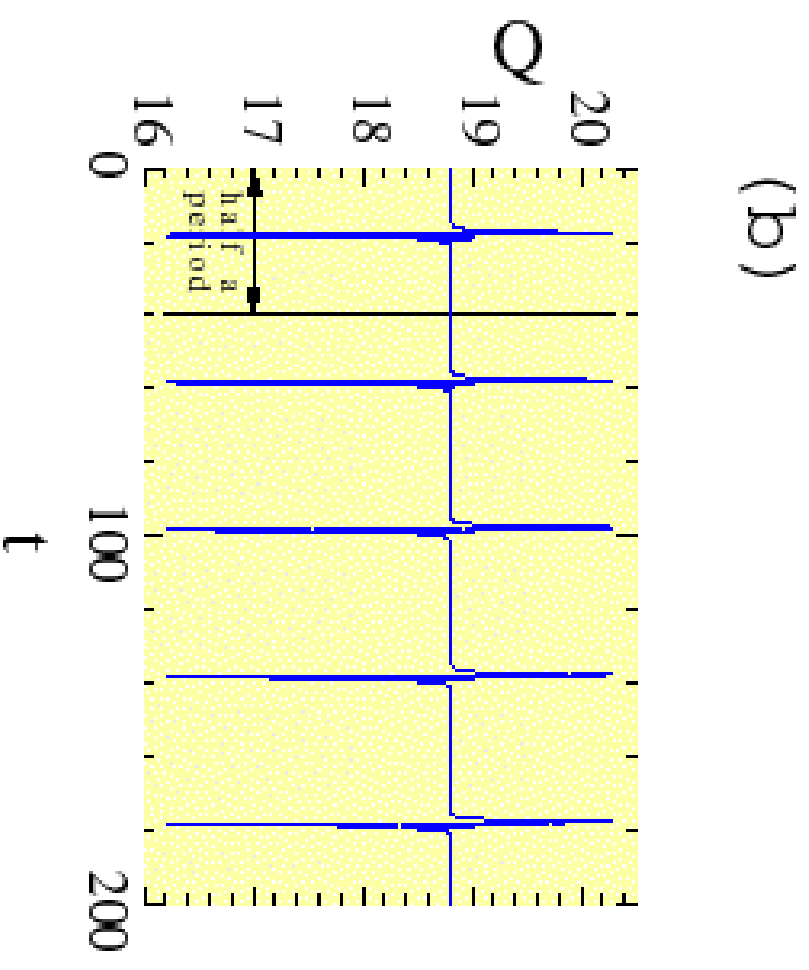
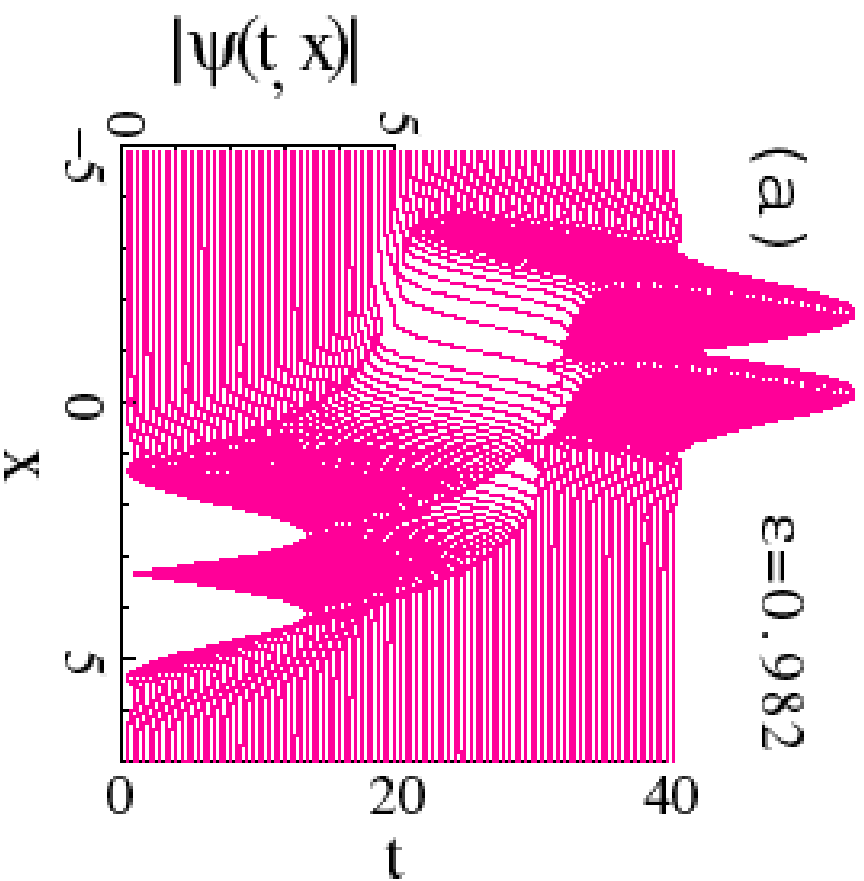
The type I double pulse solution with  $\pi$  phase difference between the solitons (blue line).

The type II (green line).

The double pulse with a  $\pi/2$  phase difference between them (red line).

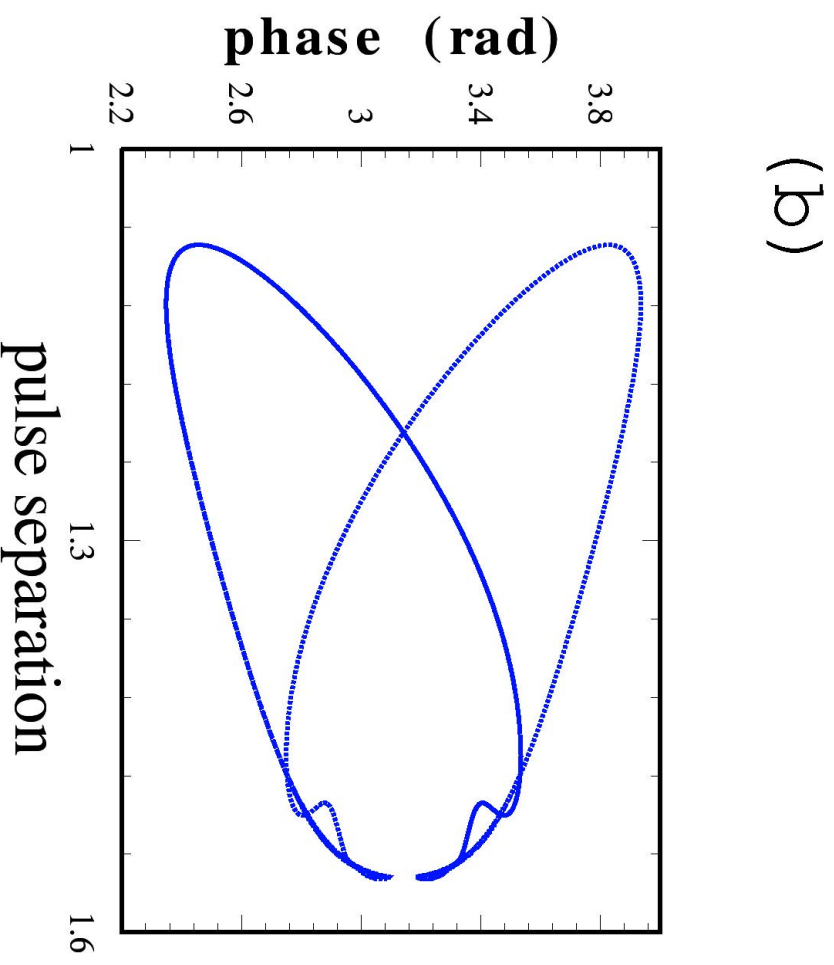
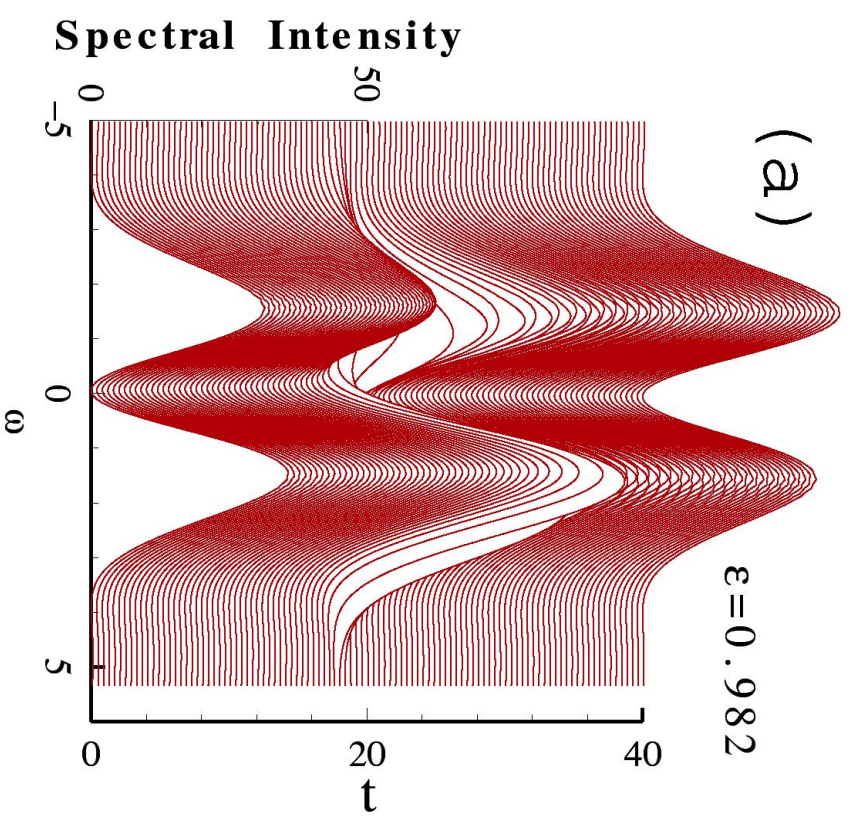
Periodic evolution of the double pulse with  $\pi$  phase difference.

## Evolution of Energy





Evolution of the spectrum of the two-pulse solution with  $\pi$  phase difference between the pulses. Periodicity.

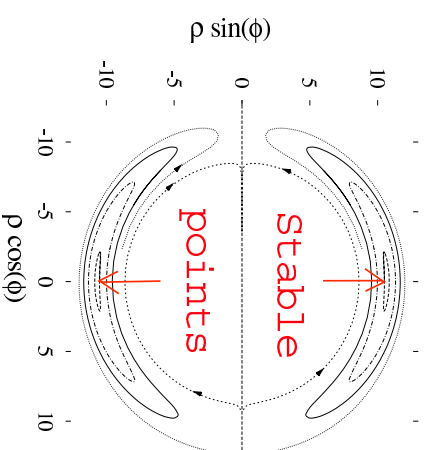
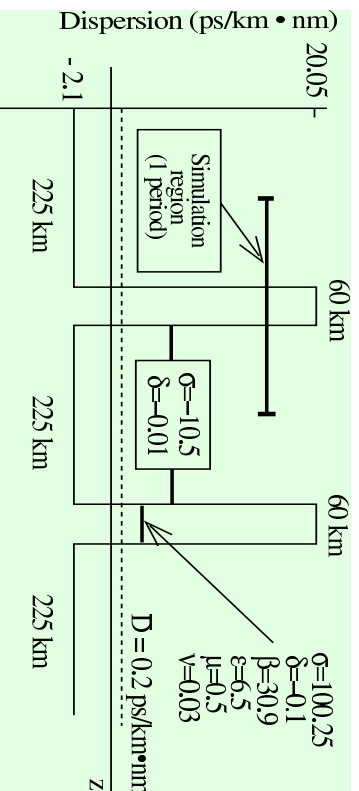


# Importance: Stable High Density Information Transmission

**Research objective:** Increasing the Capacity of High Density Optical Transmission Lines

**Means:** Controlled Pulse-pulse interaction in dispersion managed fiber system when nonlinear amplifiers are added

**Scientific Barriers:** The pulses tend to merge or change shape before arriving to the receiver. This problem for dispersion managed fiber links was not solved.



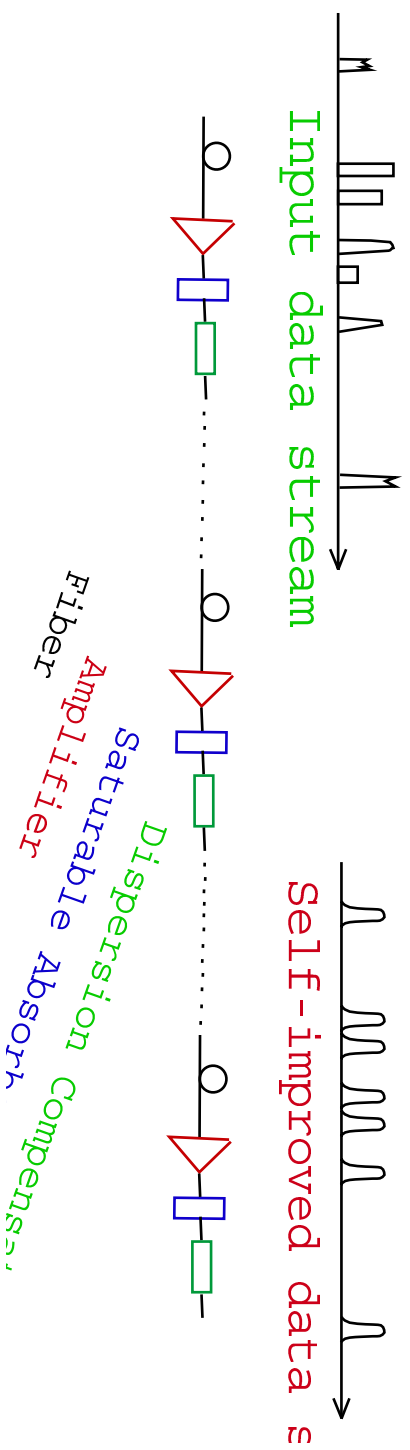
*Parameters of the optical transmission line (dispersion map) used in the simulations.*

*Interaction plane shows the existence of two stable foci or centers.*

**Main result:** Nearest pulses do not merge !

# High stability All-Optical information transmission

## Optical Transmission Link with self-adjustment



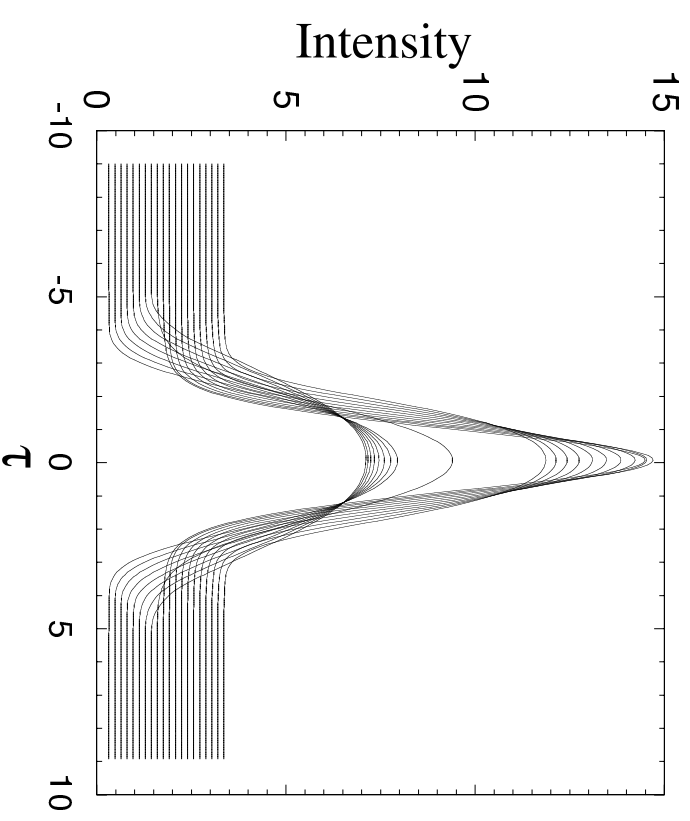
Saturable absorbers Significantly Improve Performance of an Optical Link.

This system brings to a standard the pulse shapes and distances between the pulses.

## Advantages of using nonlinear amplifiers in Dispersion - Managed Optical Transmission Systems.

Dispersion-managed optical transmission systems may greatly improve the transmission capacity of fiber links.

Periodic variations of dispersion bring the pulse back almost to its original shape after each period.



*Equation which we are solving:*

$$i \psi_z + [\sigma(z)/2] \psi_{tt} + |\psi|^2 \psi = i\delta(z) \psi + i\epsilon(z) |\psi|^2 \psi + i\beta(z) \psi_{tt} + i\mu(z) |\psi|^4 \psi - \nu(z) |\psi|^4 \psi$$

# Conclusions

The system modelled has a larger number of soliton solutions than a system modelled by the complex CGLE due to the more complicated spectral response intrinsic to the CSHE model. This is a hint for facilitating an experimental observation of composite pulses.

CSHE model admits a greater variety of soliton bound states than the CGLE model: There are three different types of stable soliton bound states with  $\pi$  or  $\pi/2$  phase difference between the solitons. These results suggest the design for an optical pulse train generator with controllable phase shift and pulse separation between the pulses. This would be an ideal source for all-optical high-bit rate transmission lines.

# Observation of Soliton Explosions

**Steven T. Cundiff**

*JILA, NIST & University of Colorado, Boulder*

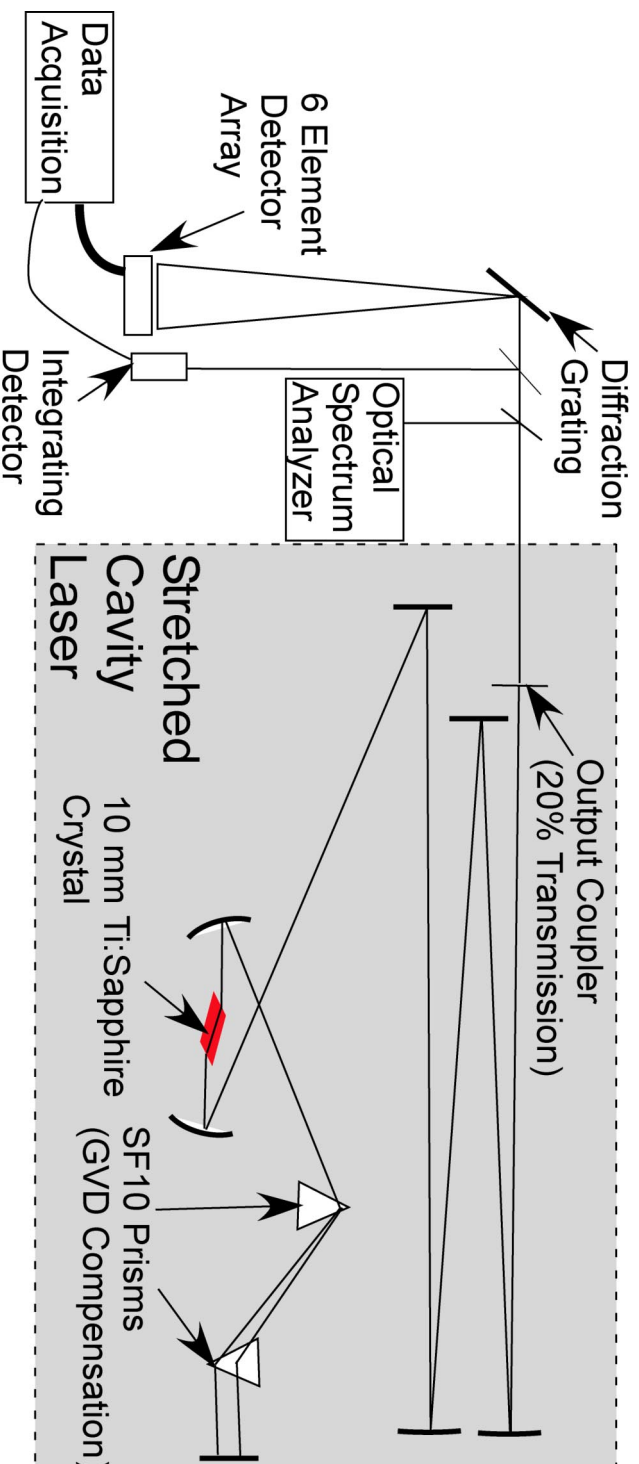
**Jose M. Soto-Crespo**

*Instituto de Óptica, CSIC, Madrid*

**Nail Akhmediev**

*Australian National University, Canberra*

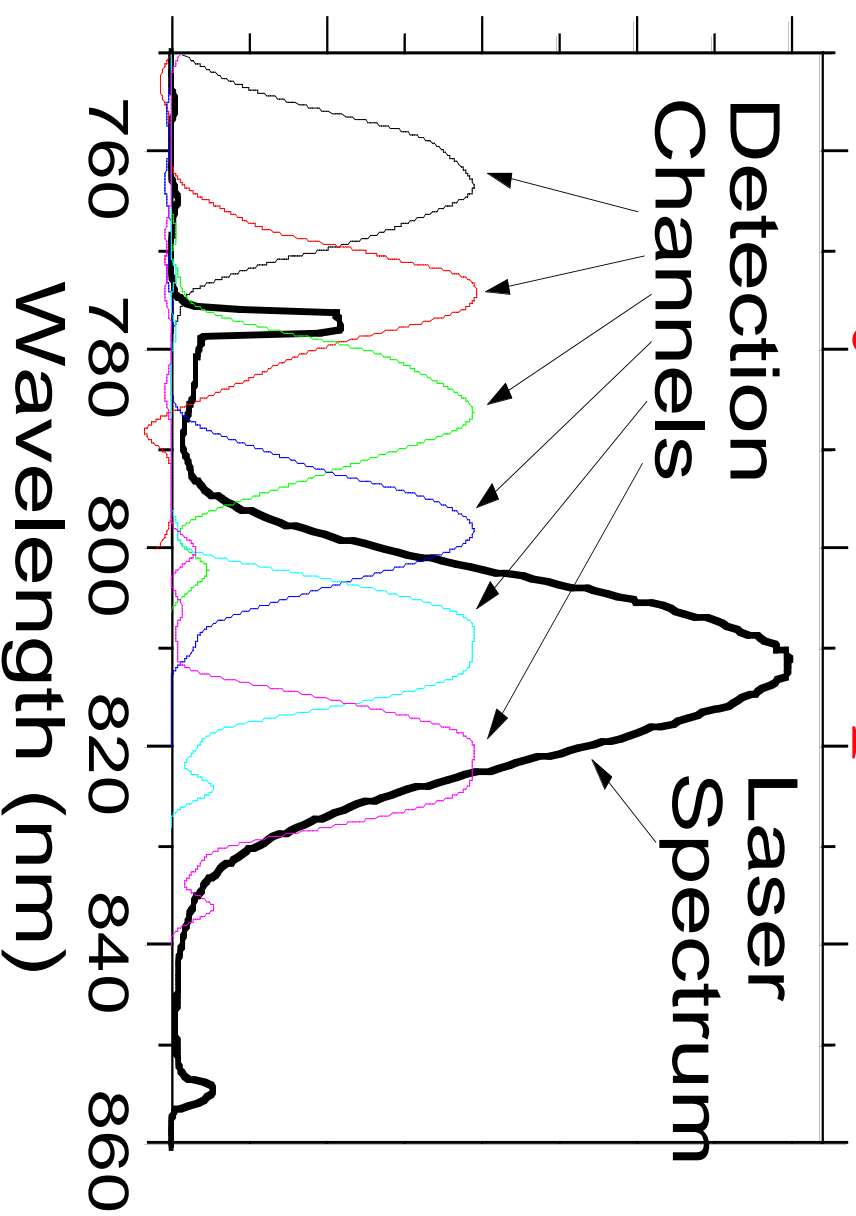
# Experimental Setup



~ 40 fs pulses

Round trip time ~ 40 ns

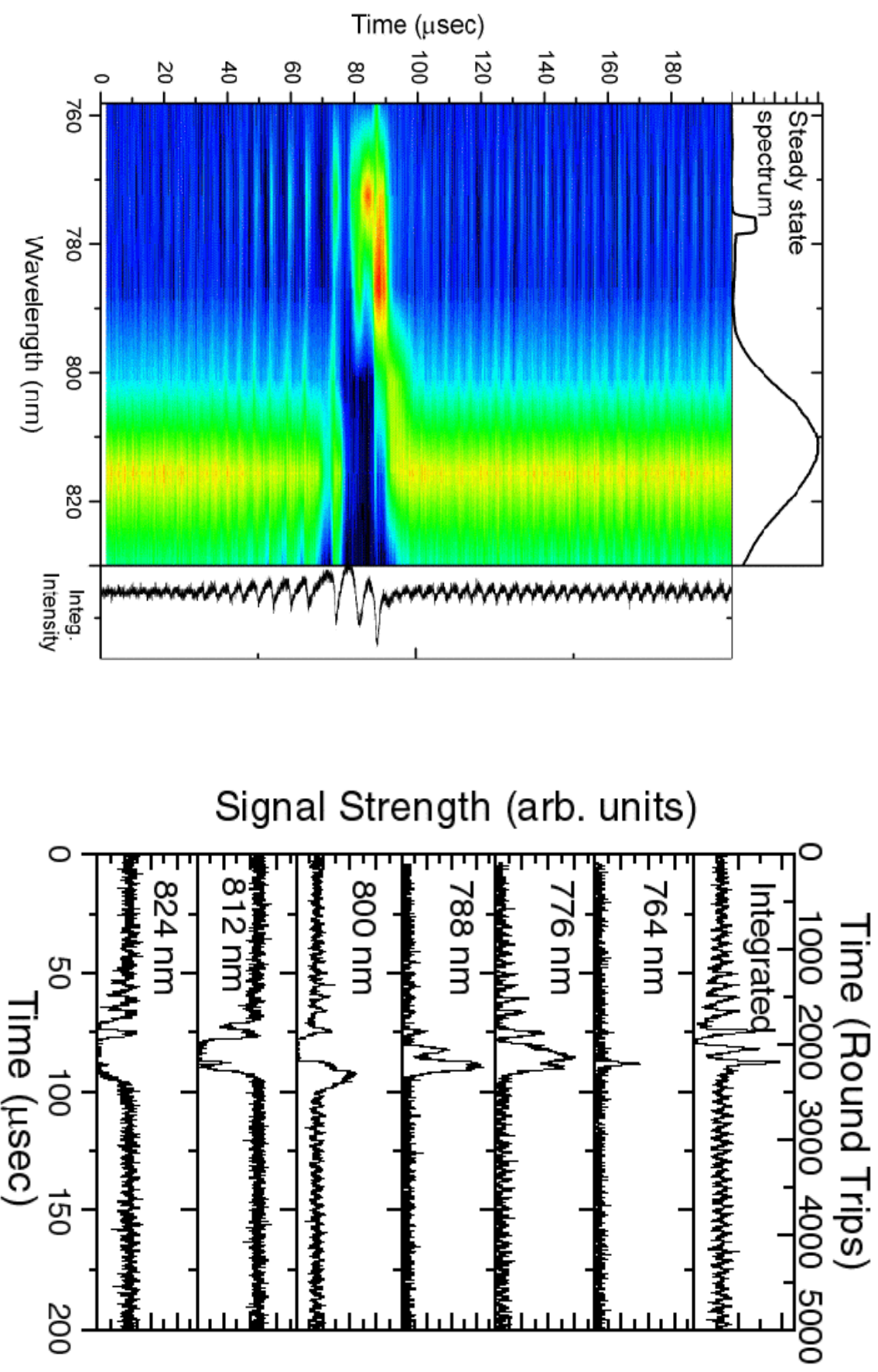
# Steady state spectrum



The spectral width of individual channels ~12 nm

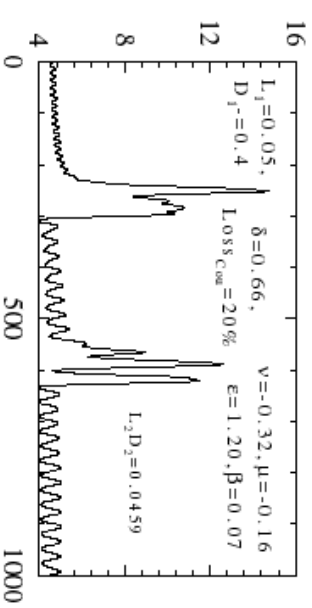


# Experiment: Solitary Explosion

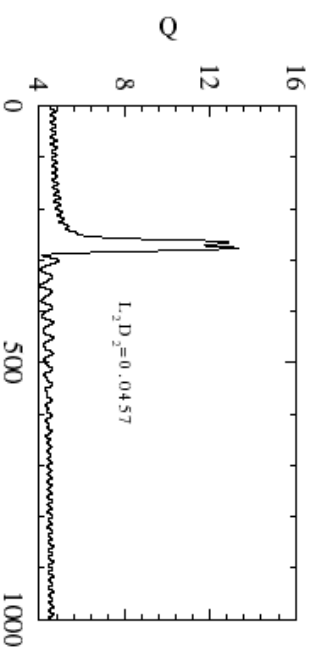


# Transition from regular pulse generation to explosions

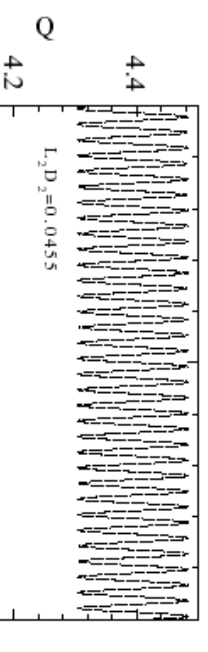
Bursts of explosions



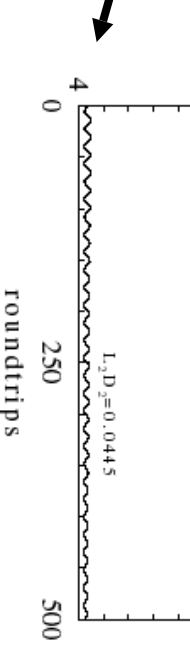
Solitary explosion



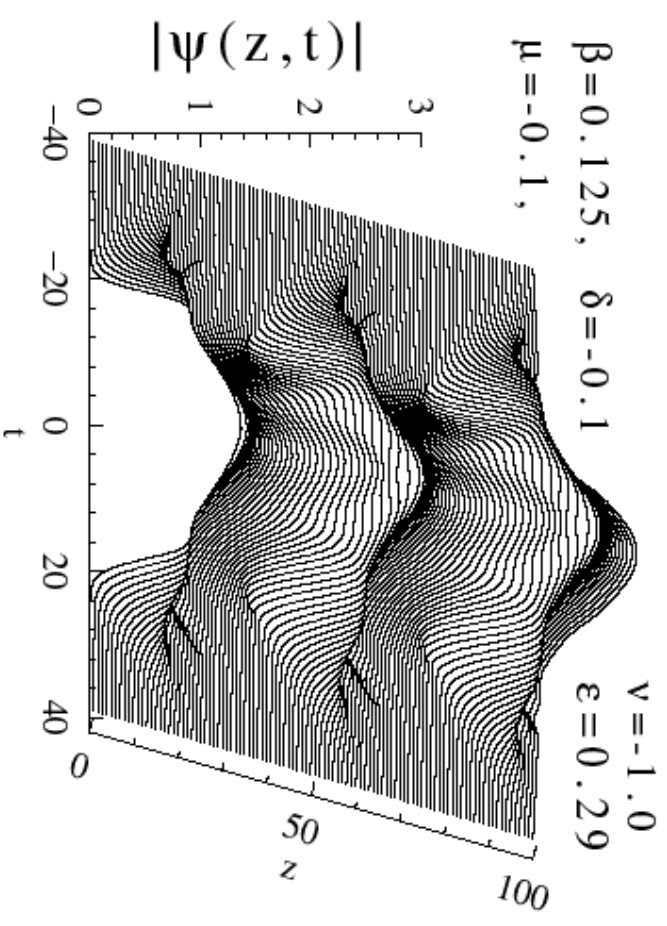
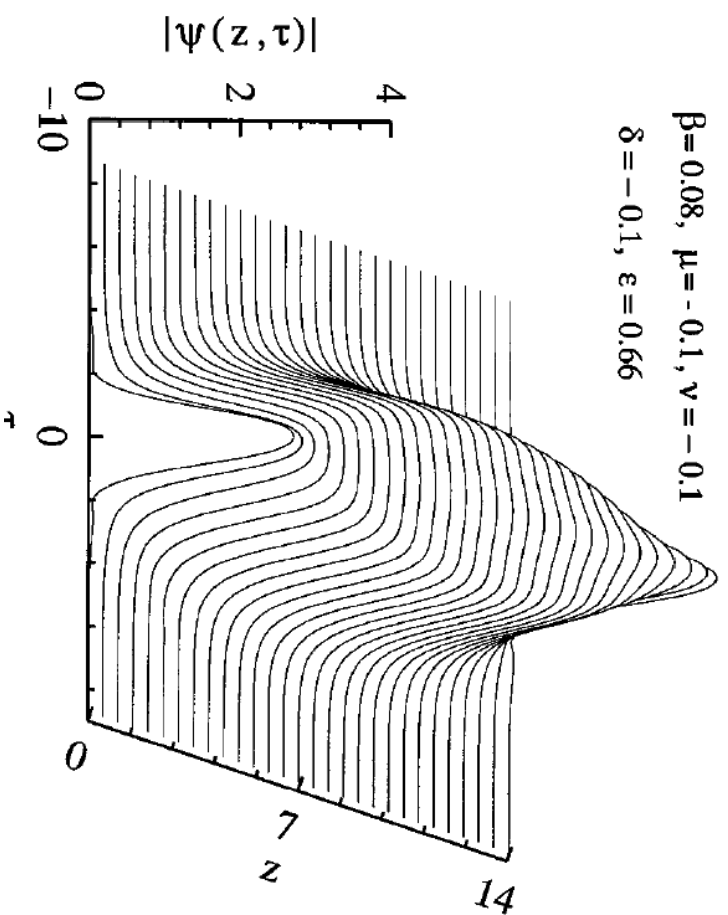
Pulsating regime



Regular pulses



# Examples of pulsating solitons



# Range of parameters

Explosions  
exist in an  
amazingly  
wide range of  
parameters

Red dot corresponds  
to the above  
example

